Hyperplane

Ling572
Advanced Statistical Methods for NLP
February 13, 2020

Points and Vectors

- A point in n-dimensional space is given by an n-tuple
  - E.g., **P=(p_i)**
  - Represents an absolute position in space

- A vector represents a magnitude and direction in space, also given by an n-tuple
  - Vectors do not have a fixed position in space
    - Can be located at any initial base point **P**
  - A vector from point **P** to point **Q** is given by:

$$v = Q - P = (q_i - p_i)$$

Vector Computation

- Vector addition: $v + w = (v_i + w_i)$
- Vector subtraction: $v - w = (v_i - w_i)$
- Length of a vector: $|v| = \sqrt{\sum_{i=1}^{n} v_i^2}$
- http://geomalgorithms.com/points_and_vectors.html

Normal Vector

- A normal vector is a vector perpendicular (i.e. orthogonal) to another object, e.g. a plane

- A unit normal vector is a vector of length 1
  - If N is normal vector, the unit normal vector is $\frac{N}{|N|}$
    - Where is lNl is the length of N

Equation for a Hyperplane

- A 3-D plane determined by normal vector N=(A,B,C) and point Q= (x0, y0, z0) is:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

- Which can be written as

$$Ax + By + Cz + D = 0 \quad \text{where} \quad D = -Ax_0 - By_0 - Cz_0$$

- Hyperplane:, $wx + d = 0$
  - Where w is a normal vector, x is any point on hyperplane
  - Separates the space into 2 half spaces:

$$wx + d < 0 \quad wx + d > 0$$

Distance from Point to Plane

- Given a plane Ax+By+Cz+D=0 and point P=(x_1,y_1,z_1), the distance from P to the plane is:

$$\frac{|Ax_1 + By_1 + Cz_1 + d|}{\sqrt{A^2 + B^2 + C^2}}$$

- More generally, distance from point x to hyperplane wx+d=0 is:

$$\frac{|wx + d|}{||w||}$$

Distance between two parallel planes

- Two planes $A_1x+B_1y+C_1z+D_1=0$ and $A_2x+B_2y+C_2z+D_2=0$ are parallel if:
  - $A_1=kA_2$ and $B_1=kB_2$ and $C_1=kC_2$

- The distance between (parallel) planes $Ax+By+Cz+D1=0$ and $Ax+By+Cz+D2=0$ is equal to the distance between a point (x_1,y_1,z_1) on one plane to the other

$$\frac{|Ax_1 + By_1 + Cz_1 + D_2|}{\sqrt{A^2 + B^2 + C^2}} = \frac{|D_2 - D_1|}{\sqrt{A^2 + B^2 + C^2}}$$