Hyperplane

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Advanced Statistical Methods for NLP
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Points and Vectors

- A point in n-dimensional space is given by an n-tuple
 - E.g., $P=(p_i)$
 - Represents an absolute position in space

- A vector represents a magnitude and direction in space, also given by an n-tuple
 - Vectors do not have a fixed position in space
 - Can be located at any initial base point P
 - A vector from point P to point Q is given by:

$$V=Q-P=(q_i-p_i)$$

Vector Computation

Vector addition:

$$V+W=(V_i+W_i)$$

Vector subtraction:

$$V-W=(V_i-W_i)$$

• Length of a vector:

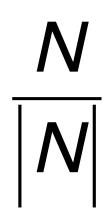
$$|\mathbf{V}| = \sqrt{\sum_{i=1}^{n} \mathbf{V}_{i}^{2}}$$

http://geomalgorithms.com/points_and_vectors.html

Normal Vector

 A normal vector is a vector perpendicular (i.e. orthogonal) to another object, e.g. a plane

- A unit normal vector is a vector of length 1
 - If N is normal vector, the unit normal vector is
 - Where is INI is the length of N



Equation for a Hyperplane

A 3-D plane determined by normal vector N=(A,B,C) and point Q= (x0, y0, z0) is:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$$

Which can be written as

$$Ax + By + Cz + D = 0$$
 where $D = -Ax_0 - By_0 - Cz_0$

- Hyperplane:, wx + d = 0
 - Where w is a normal vector, x is any point on hyperplane
 - Separates the space into 2 half spaces:

$$wx + d < 0$$
 $wx + d > 0$

Distance from Point to Plane

Given a plane Ax+By+Cz+D=0 and point P=(x₁,y₁,z₁), the distance from P to the plane is:

$$|Ax_1 + By_1 + Cz_1 + d|$$
 $\sqrt{A^2 + B^2 + C^2}$

More generally, distance from point x to hyperplane wx+d=0 is:

$$|wx+d|$$
 $|w|$

Distance between two parallel planes

- Two planes $A_1x+B_1y+C_1z+D_1=0$ and $A_2x+B_2y+C_2z+D_2=0$ are parallel if:
 - $A_1=kA_2$ and $B_1=kB_2$ and $C_1=kC_2$
- The distance between (parallel) planes Ax+By+Cz+D1=0 and Ax+By+Cz+D2=0 is equal to the distance between a point (x₁,y₁,z₁) on one plane to the other

$$\frac{|Ax_1 + By_1 + Cz_1 + D_2|}{\sqrt{A^2 + B^2 + C^2}} = \frac{|D_2 - D_1|}{\sqrt{A^2 + B^2 + C^2}}$$