Gradient Descent; Word Vectors

LING 575K Deep Learning for NLP

Shane Steinert-Threlkeld

March 30 2022

Announcements

- Office hours:
  - Shane: Mon 3-5PM
  - Agatha:
    - Monday 10-11AM
    - Wednesday 3:30-4:30PM
      - https://washington.zoom.us/my/agathadowney
      - in-person by appointment

Today’s Plan

- Terminology / Notation
- Gradient Descent
- Word Vectors, intro
- Homework 1

Basic Terminology / Notation

Supervised Learning

Supervised Learning

- Given: a dataset $\mathscr{D} = \{(x_1, y_1), \ldots, (x_n, y_n)\}$
  - $x_i \in X$: input for i-th example
  - $y_i \in Y$: output for i-th example

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- For example:
  - Sentiment analysis:
    - Input: bag of words representation of "This movie was great."
    - Output: 4 [on a scale 1-5]
  - Language modeling:
    - Input: "This movie was"
    - Output: "great"

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- Goal: *learn* a function $f : X \to Y$ which:
  - "Does well" on the given data $\mathscr{D}$
  - Generalizes well to unseen data

Parameterized Functions

- A learning algorithm searches for a function f amongst a space of possible functions
- Parameters define a family of functions
  - θ: general symbol for parameters
  - $\hat{y} = f(x;\theta)$: input x, parameters θ; model/function output $\hat{y}$
- Example: the family of linear functions $f(x) = mx + b$
  - $\theta = \{m, b\}$
- Later: neural network architecture defines the family of functions

Loss Minimization

- General form of optimization problem
- $\mathscr{L}(\hat{Y}, Y)$: loss function ("objective function"); $\mathscr{L}(\hat{Y}, Y) = \frac{1}{|Y|} \sum_i \ell(\hat{y}(x_i), y_i)$
  - How "close" are the model's outputs to the true outputs
  - $\ell(\hat{y}, y)$: local (per-instance) loss, averaged over training instances
  - More later: depends on the particular task, among other things
- View the loss *as a function of the model's parameters*

$$\mathscr{L}(\theta) := \mathscr{L}(\hat{Y}, Y) = \mathscr{L}(f(X; \theta), Y)$$

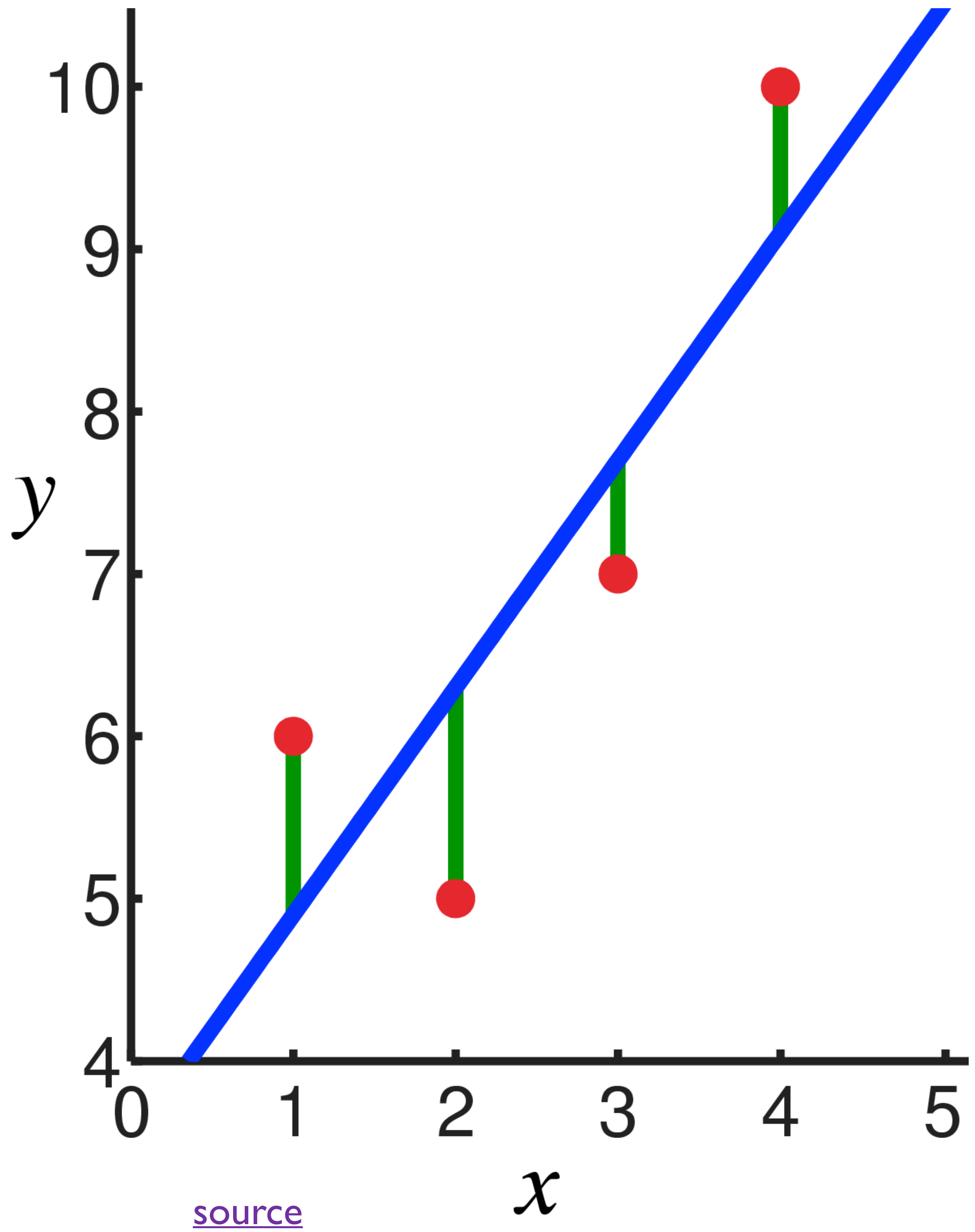
Loss Minimization

- The optimization problem:

$$\theta^* = \arg\min_{\theta} \mathcal{L}(\theta)$$

- Example: (least-squares) linear regression
  - $\ell(\hat{y}, y) = (\hat{y} - y)^2$

$$m^*, b^* = \arg\min_{m,b} \sum_i ((mx_i + b) - y_i)^2$$



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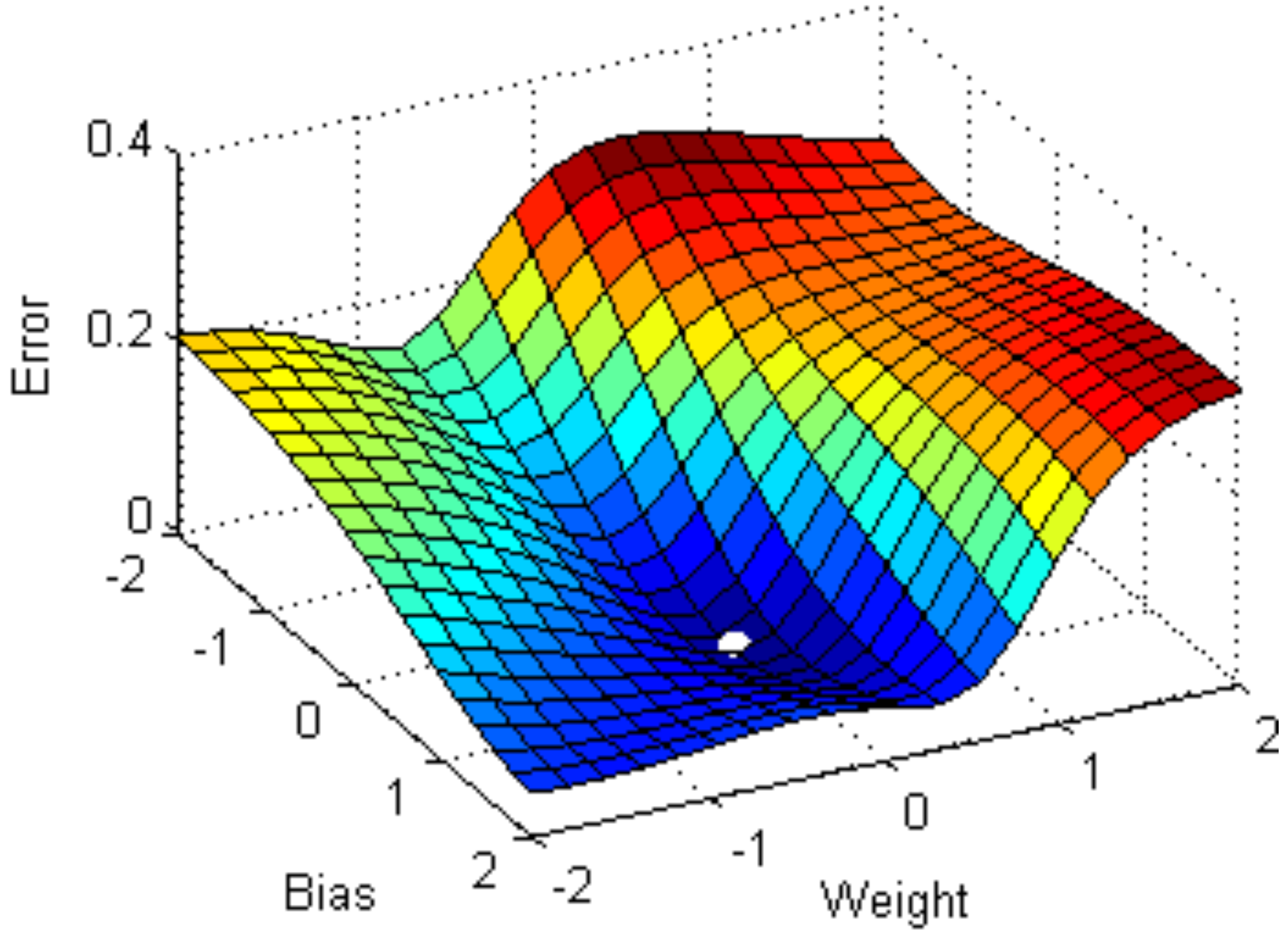
Learning: (Stochastic) Gradient Descent

Gradient Descent: Basic Idea



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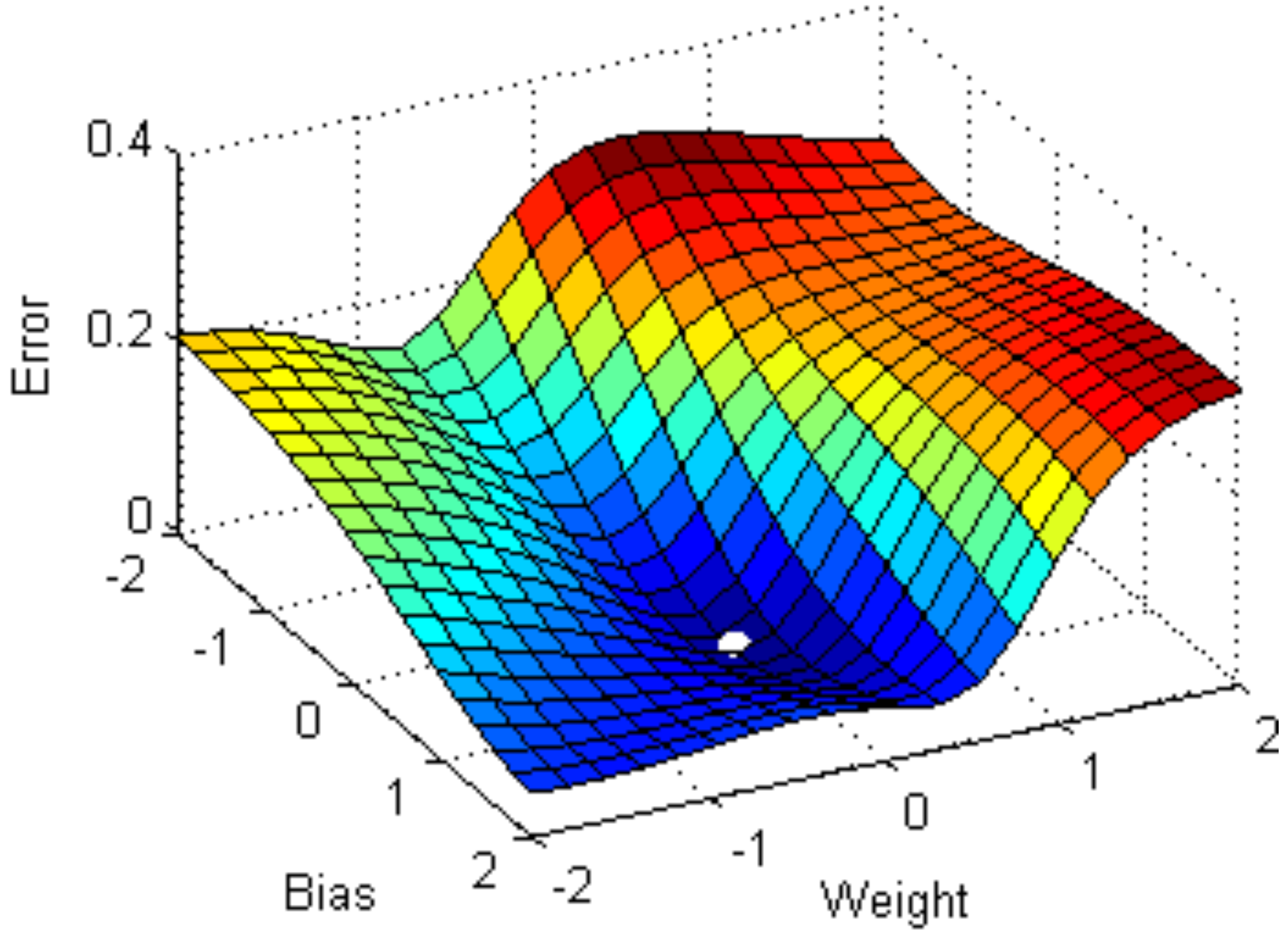

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Gradient Descent: Basic Idea

- The *gradient* of the loss w/r/t parameters tells which direction in parameter space to "walk" to make the loss smaller (i.e. to improve model outputs)
- Guaranteed to work in linear model case
  - Can get stuck in local minima for non-linear functions, like NNs
  - [More precisely: if loss is a *convex* function of the parameters, gradient descent is guaranteed to find an optimal solution. For non-linear functions, the loss will generally *not* be convex.]

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$$\frac{\partial f}{\partial y} = 20x^3y + 15xy^2 + 1$$

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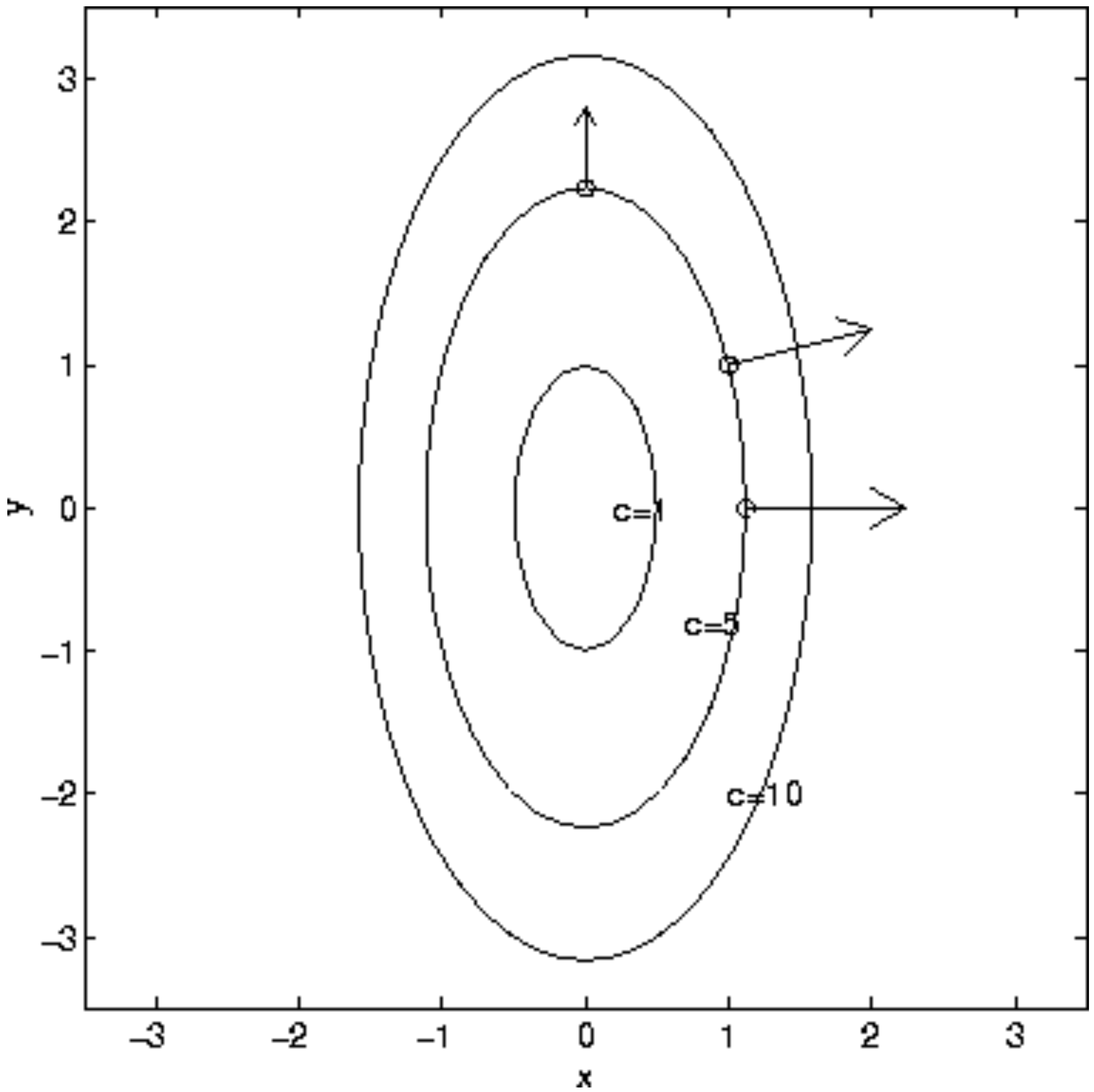
- The gradient is perpendicular to the *level curve* at a point
- The gradient points in the direction of greatest rate of increase of f

Gradient and Level Curves



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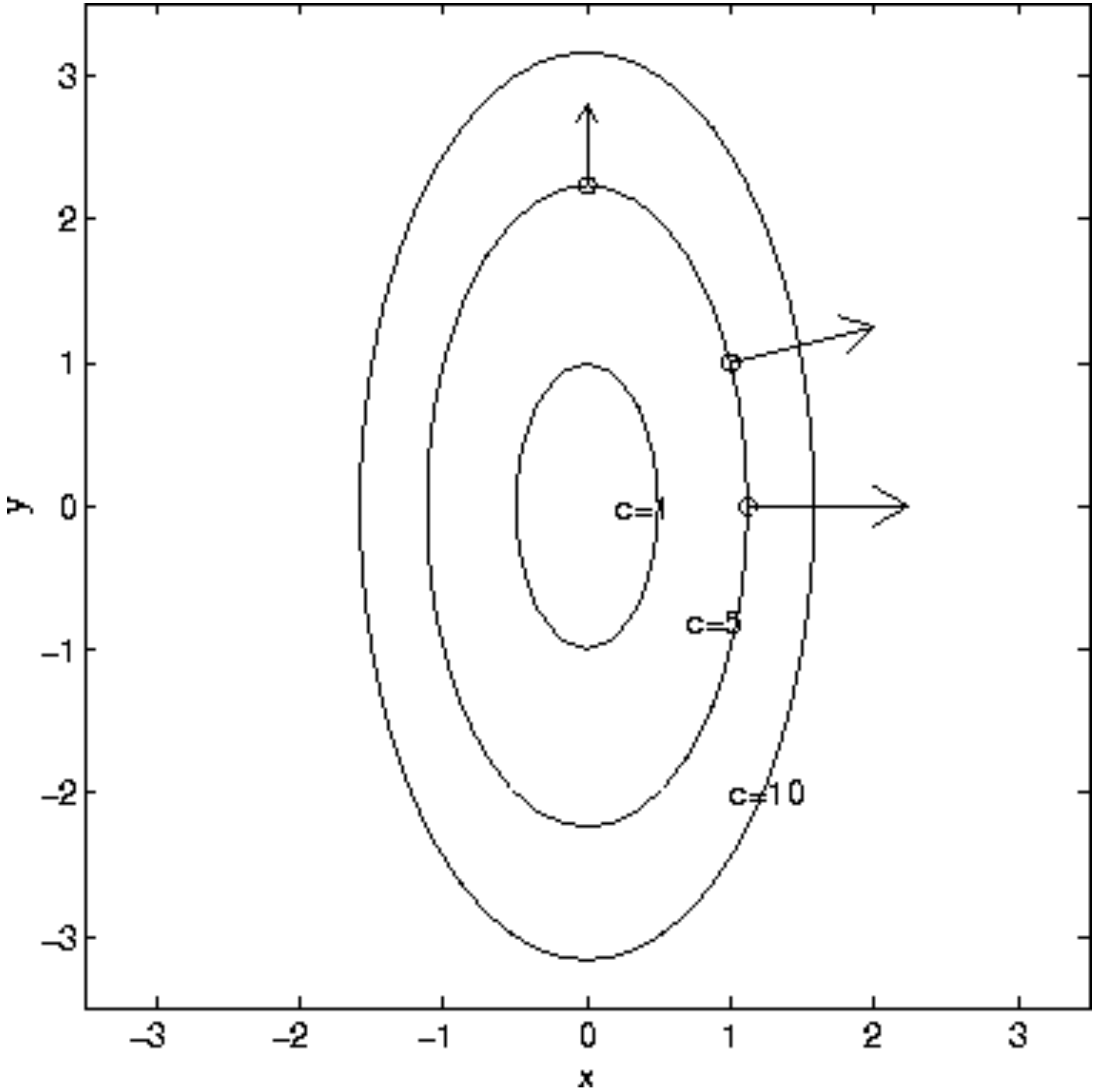

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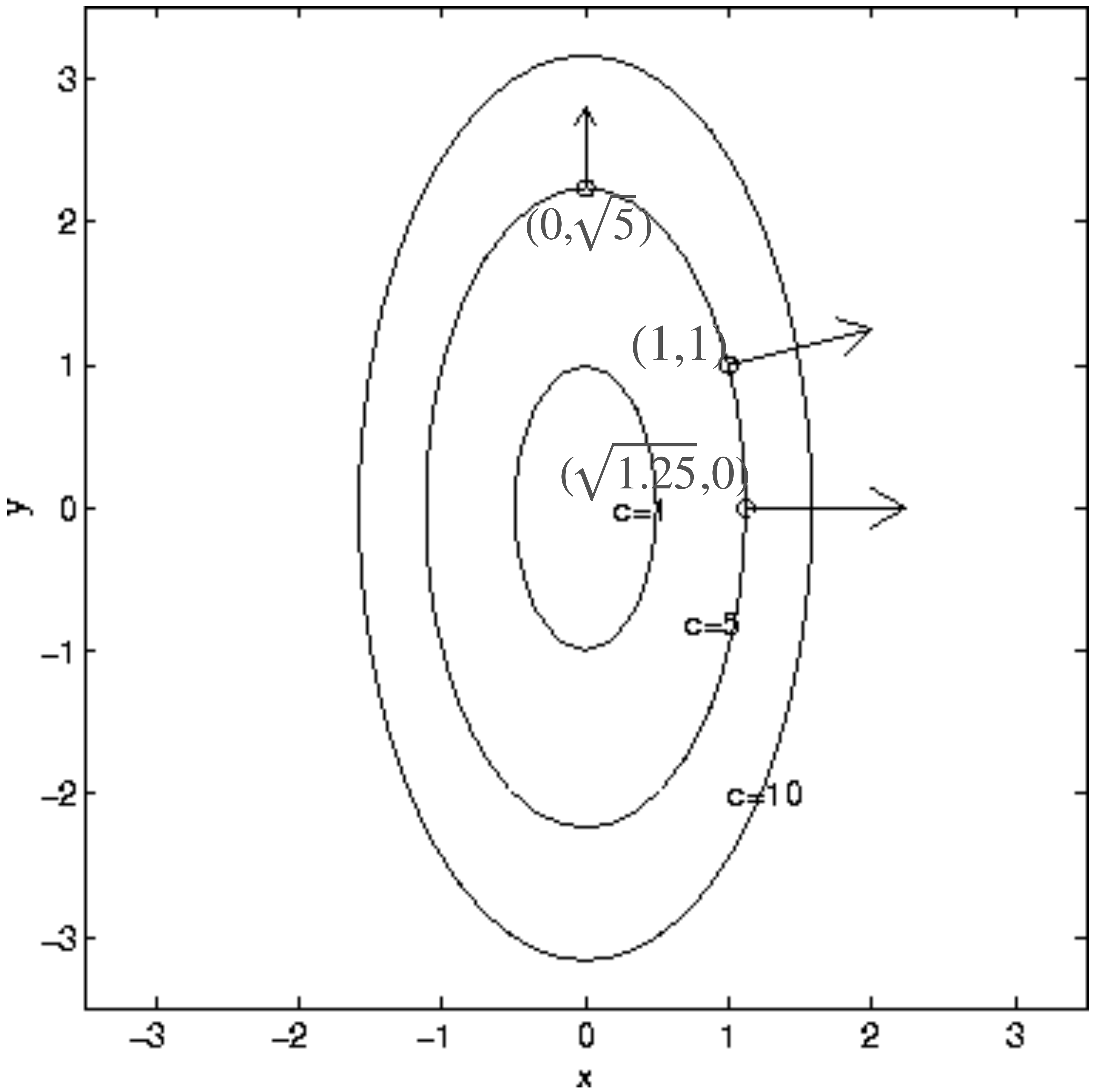

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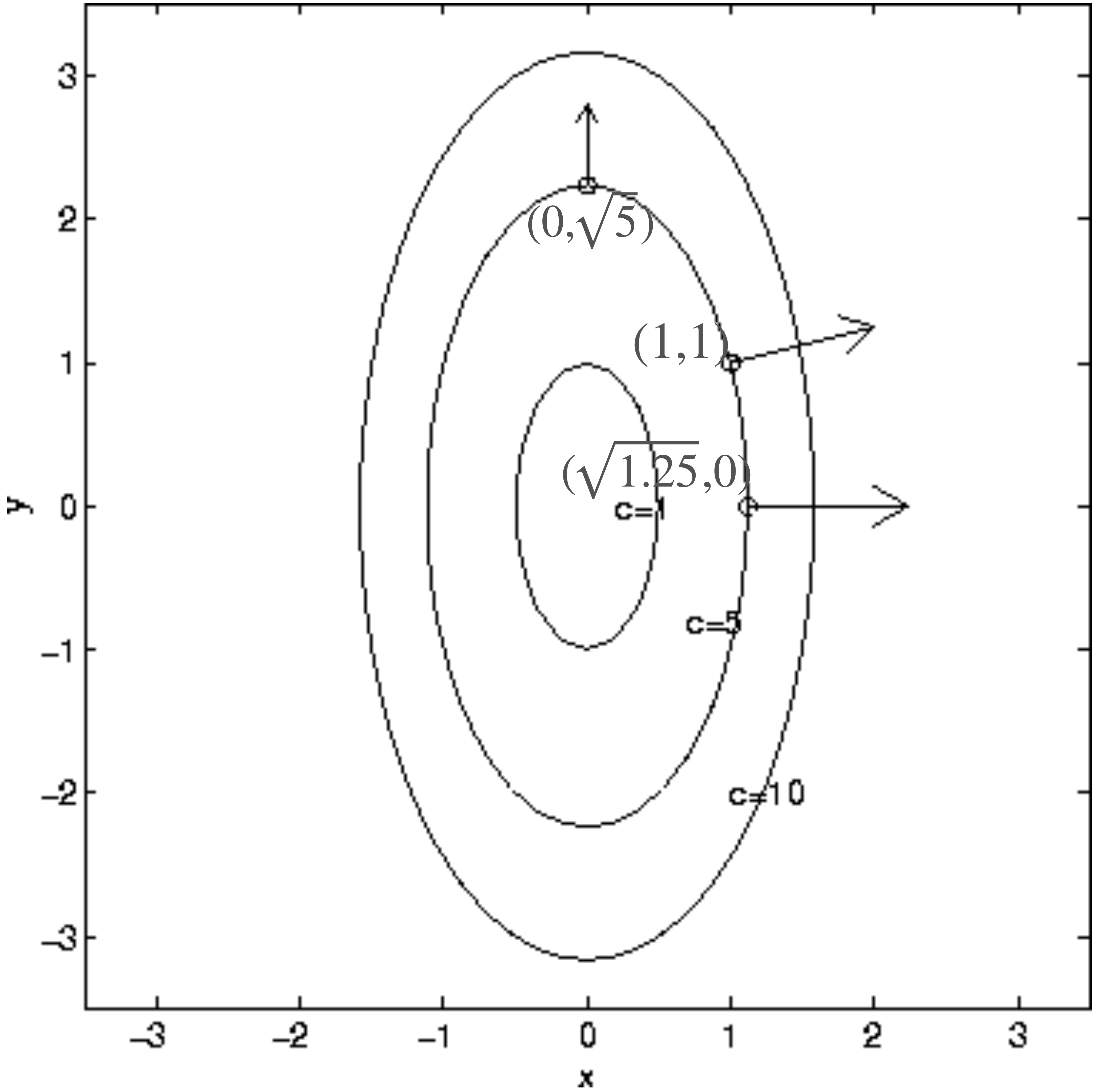

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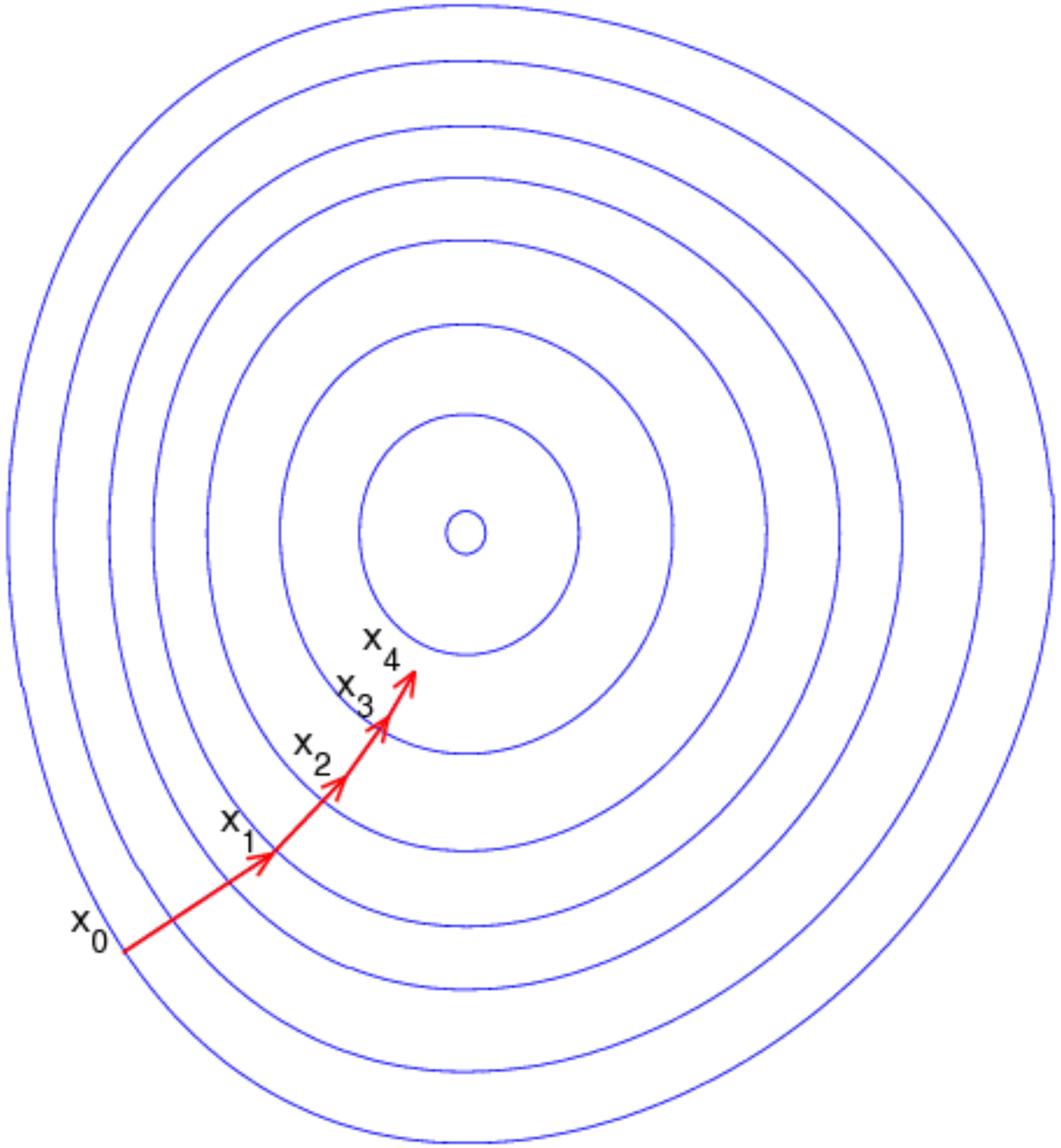
Q: what are the actual gradients at those points?

Gradient Descent and Level Curves



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Gradient Descent Algorithm

- Initialize θ_0
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$$\theta_{n+1} = \theta_n - \alpha \nabla \mathscr{L}(\hat{Y}(\theta_n), Y)$$

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Learning rate

Gradient Descent Algorithm

- Initialize θ_0
- Repeat until convergence:

$$\theta_{n+1} = \theta_n - \underbrace{\alpha}_{\text{Learning rate}} \nabla \mathscr{L}(\hat{Y}(\theta_n), Y)$$

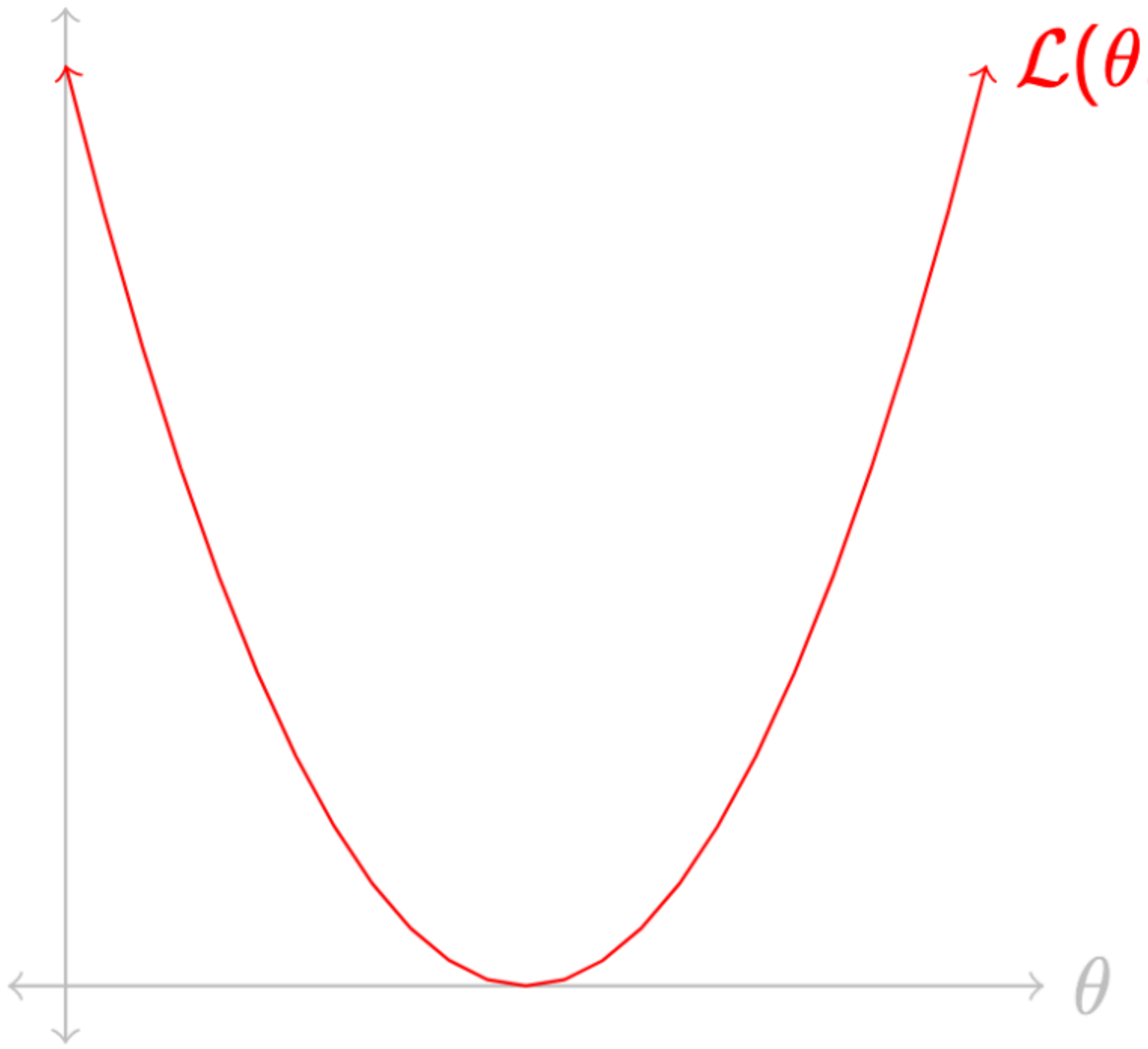
- High learning rate: big steps, may bounce and “overshoot” the target
- Low learning rate: small steps, smoother minimization of loss, but can be slow

Gradient Descent: Minimal Example

- Task: predict a target/true value $y = 2$
- "Model": $\hat{y}(\theta) = \theta$
  - A single parameter: the actual guess
- Loss: Euclidean distance

$$\mathcal{L}(\hat{y}(\theta), y) = (\hat{y} - y)^2 = (\theta - y)^2$$

Gradient Descent: Minimal Example



$$\frac{\partial}{\partial \theta}\mathcal{L}(\theta, y) = 2(\theta - y)$$

$$\theta_{t+1} = \theta_t - \alpha \cdot \frac{\partial}{\partial \theta}\mathcal{L}(\theta, y)$$

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  - Break the data into “mini-batches”: small chunks of the data
  - Compute gradients and update parameters for each batch
  - Mini-batch of size 1 = single example = stochastic gradient descent
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- Epoch: one pass through the whole training data

Stochastic Gradient Descent

```
initialize parameters / build model

for each epoch:

 data = shuffle(data)
 batches = make_batches(data)

 for each batch in batches:

  outputs = model(batch)
  loss = loss_fn(outputs, true_outputs)
  compute gradients
  update parameters
```

Word Vectors, Intro

Distributional Similarity

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  - We make ***tezgüino*** from corn.
- Tezguino; corn-based alcoholic beverage. (From *Lin, 1998a*)

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- How can we represent the “company” of a word?
- How can we make similar words have similar representations?

Why use word vectors?

- With words, a feature is a word identity
  - Feature 5: 'The previous word was "terrible"'
  - requires exact same word to be in training and test
  - **One-hot vectors:**
    - “terrible”: [0 0 0 0 0 0 1 0 0 0 … 0]
    - Length = size of vocabulary
  - All words are as different from each other
    - e.g. “terrible” is as different from “bad” as from “awesome”

Why use word vectors?

- With embeddings (= vectors):
  - Feature is a word vector
  - 'The previous word was vector [35,22,17, …]
  - Now in the test set we might see a similar vector [34,21,14, …]
  - We can generalize to similar but unseen words!

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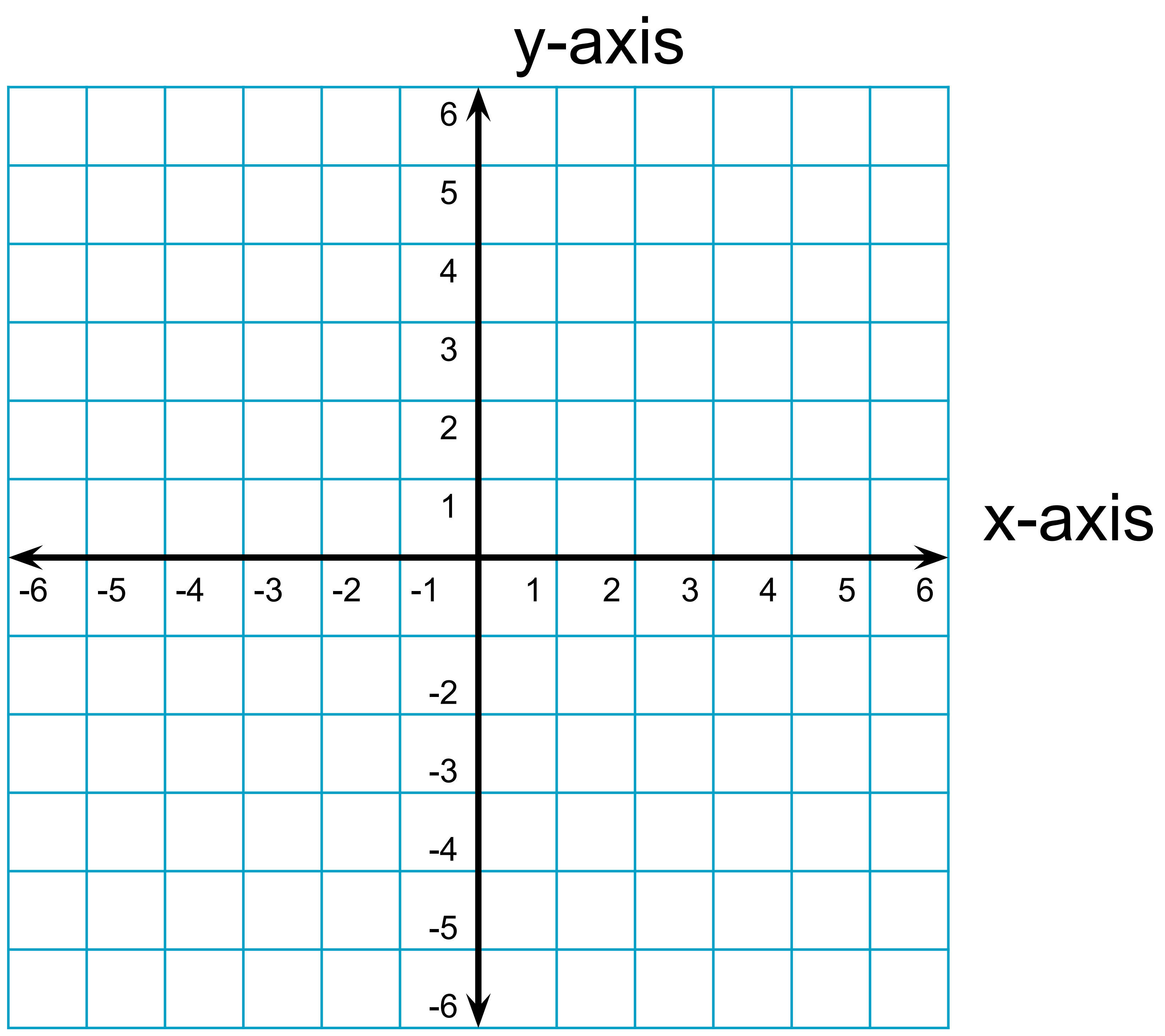

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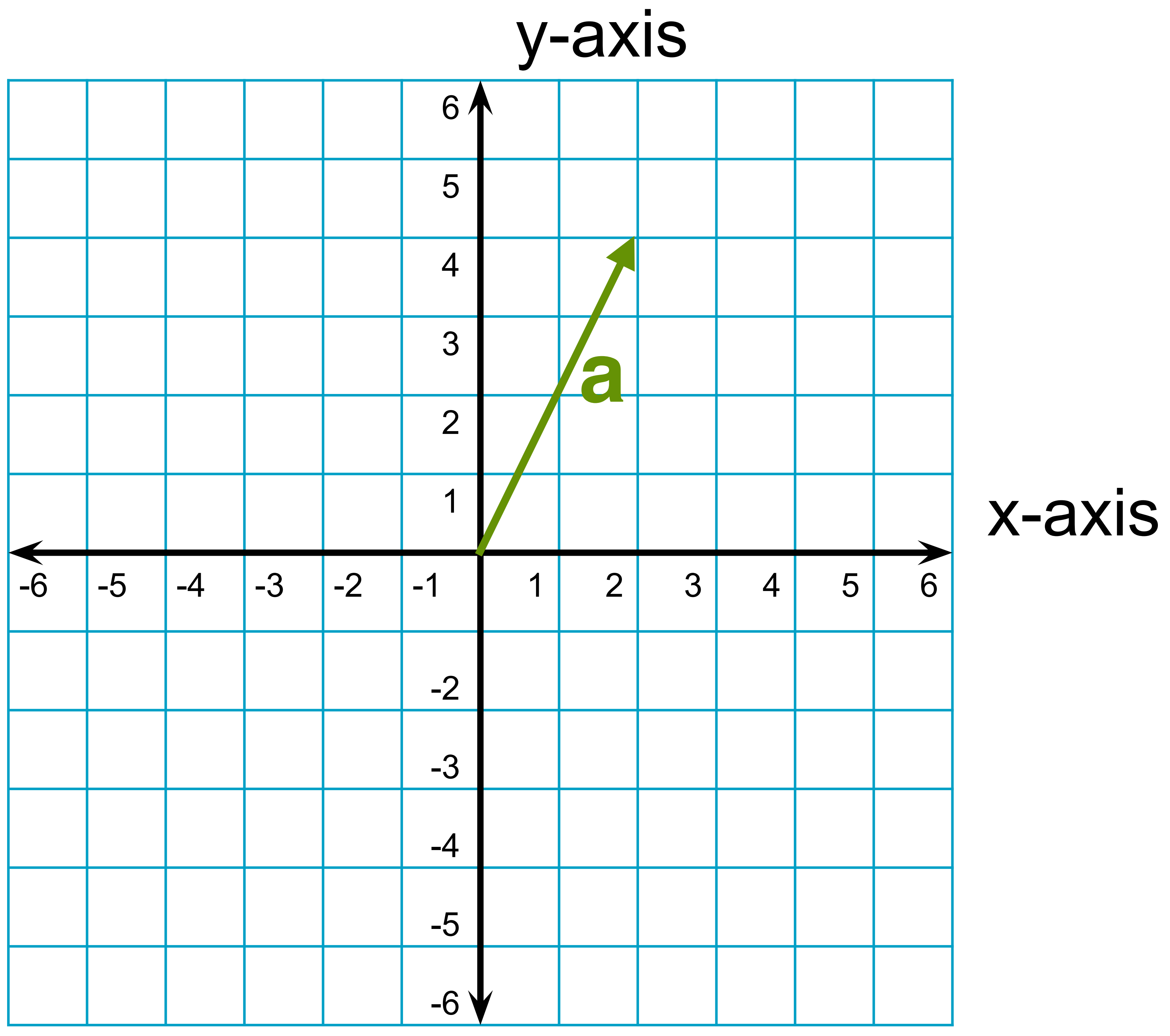

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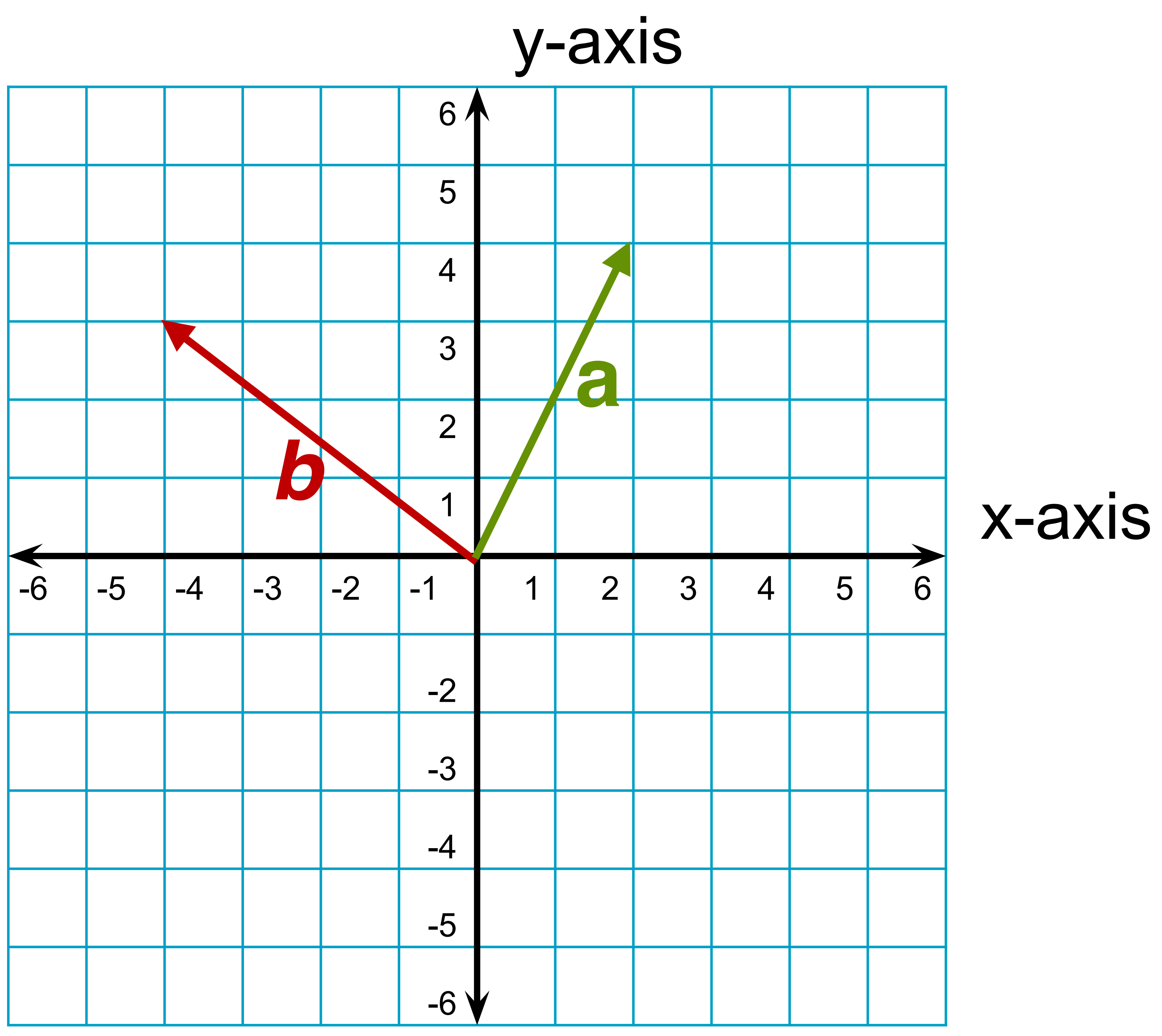

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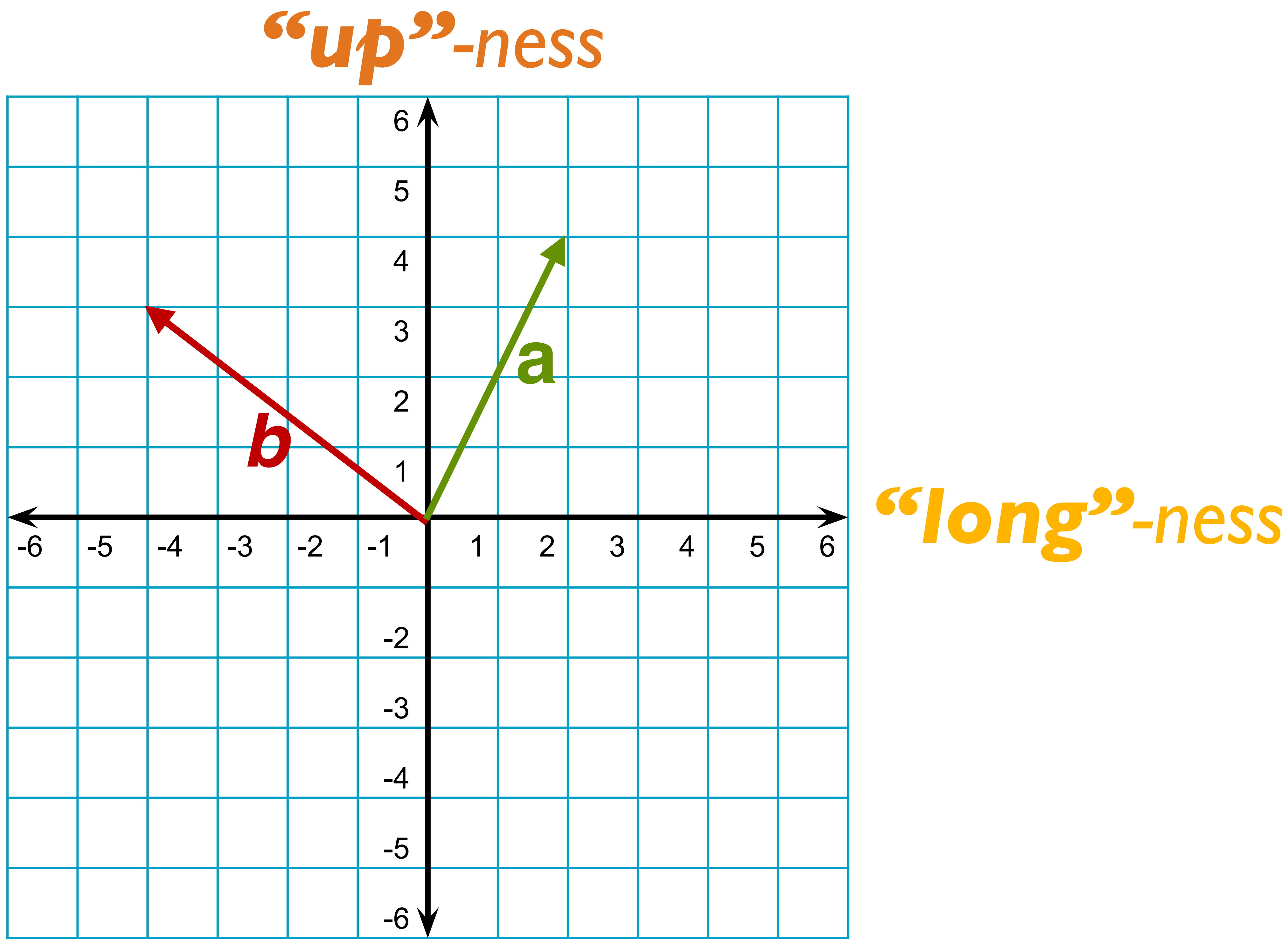

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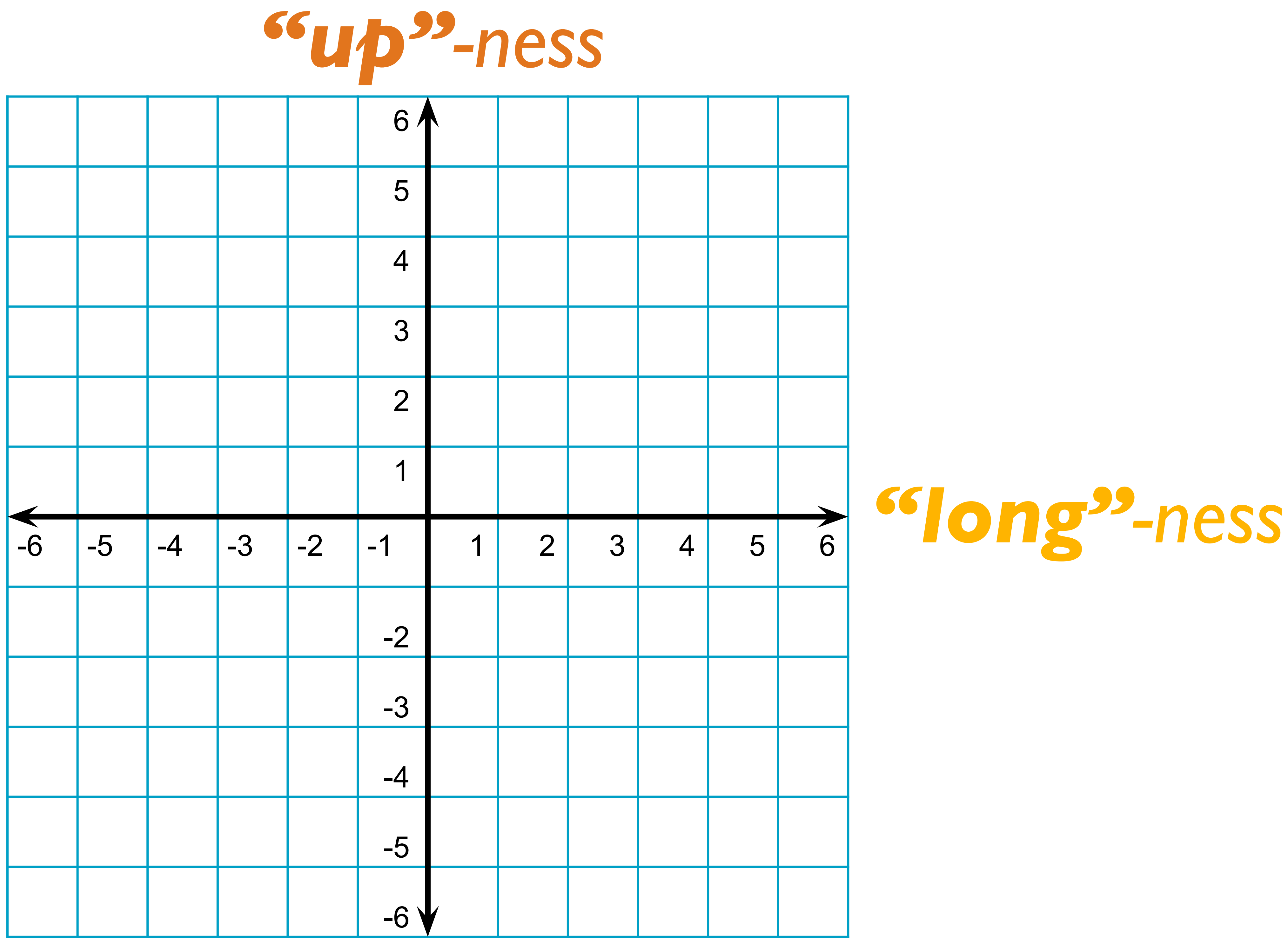

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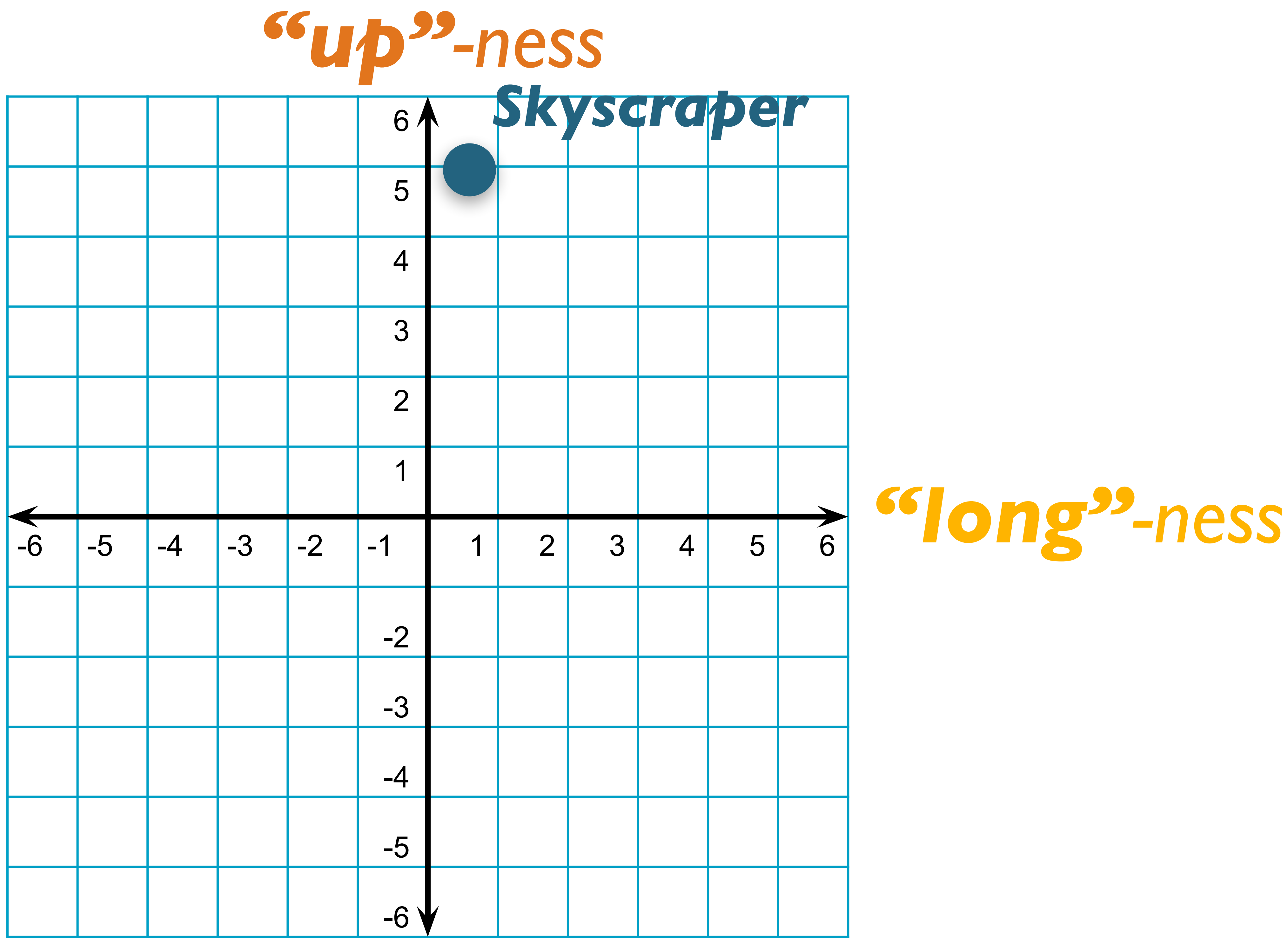

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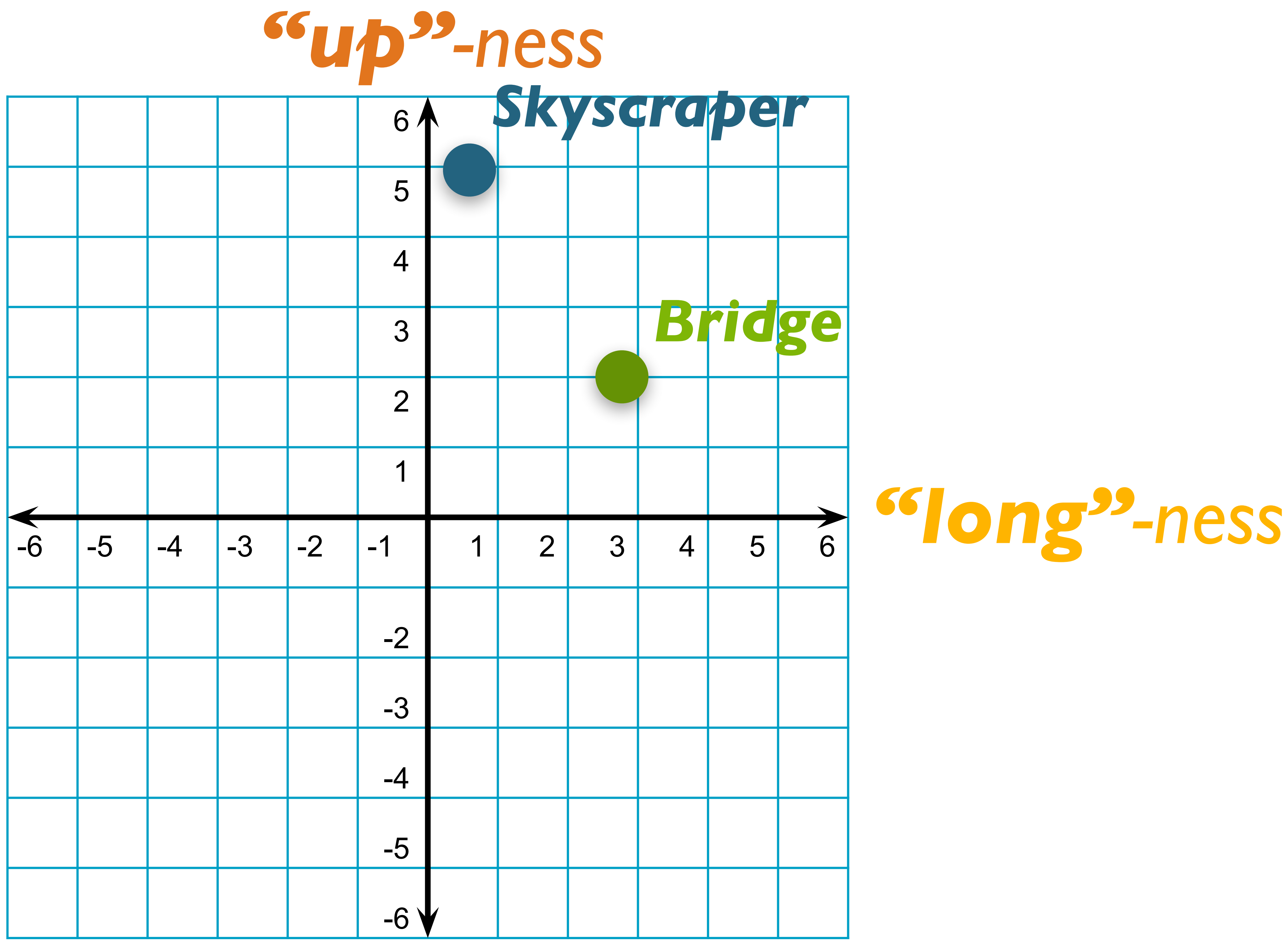

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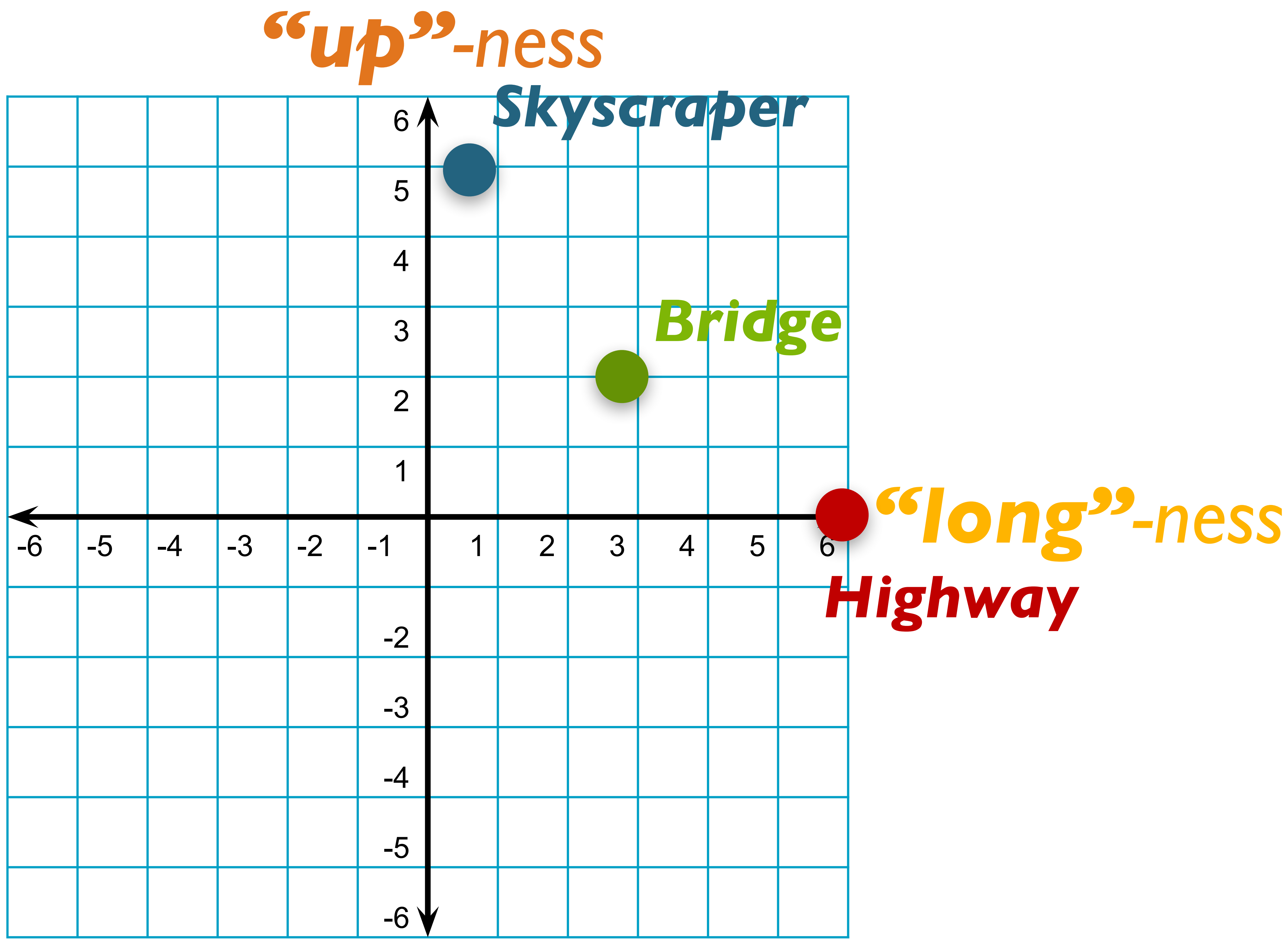

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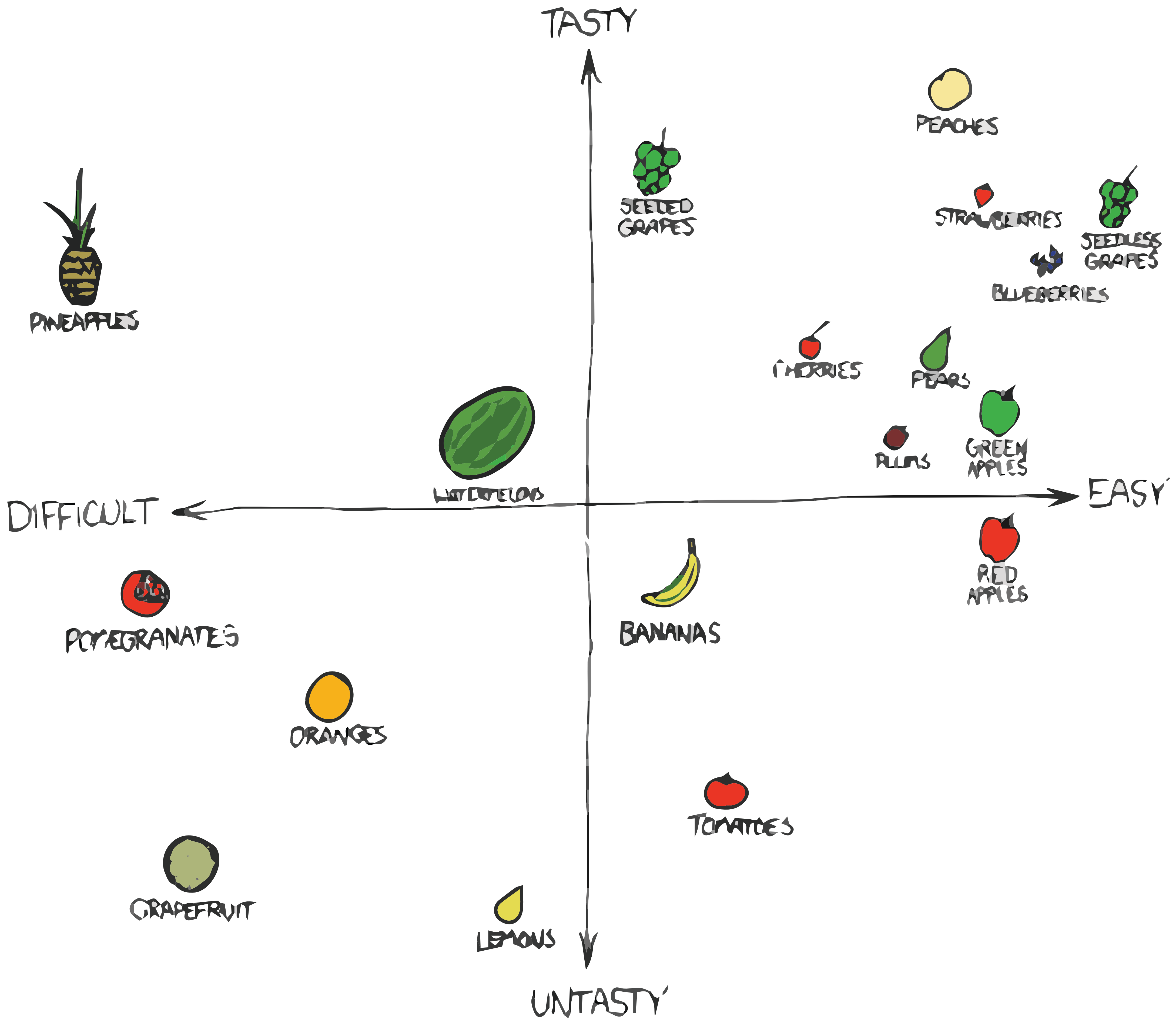
xkcd.com/388



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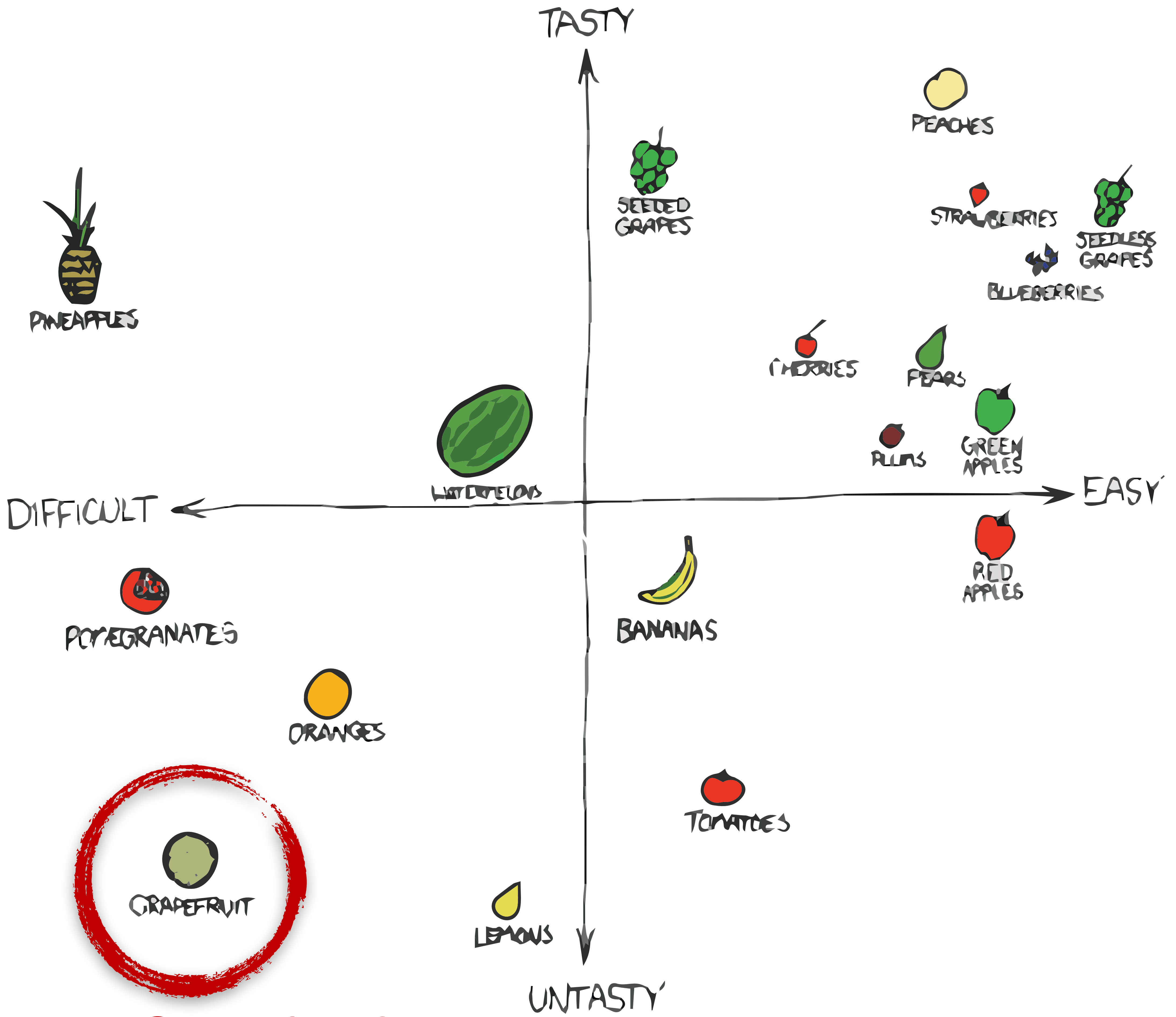

WTF, Grapefruit?

Basic vector operations

- Addition: $\mathbf{x} + \mathbf{y} = \langle \mathbf{x}_0 + \mathbf{y}_0, \ldots, \mathbf{x}_n + \mathbf{y}_n \rangle$
- Subtraction: $\mathbf{x} - \mathbf{y} = \langle \mathbf{x}_0 - \mathbf{y}_0, \ldots, \mathbf{x}_n - \mathbf{y}_n \rangle$
- Scalar multiplication: $k\mathbf{x} = \langle k\mathbf{x}_0, \ldots, k\mathbf{x}_n \rangle$
- Length: $||\mathbf{x}|| = \sqrt{\sum_i \mathbf{x}_i^2}$

Vector Distances: Manhattan & Euclidean

- **Manhattan Distance** $$d_{\text{manhattan}}(x, y) = \sum_i |x_i - y_i|$$
  - (Distance as cumulative horizontal + vertical moves)
- **Euclidean Distance**

$$d_{\text{euclidean}}(x, y) = \sum_i (x_i - y_i)^2$$

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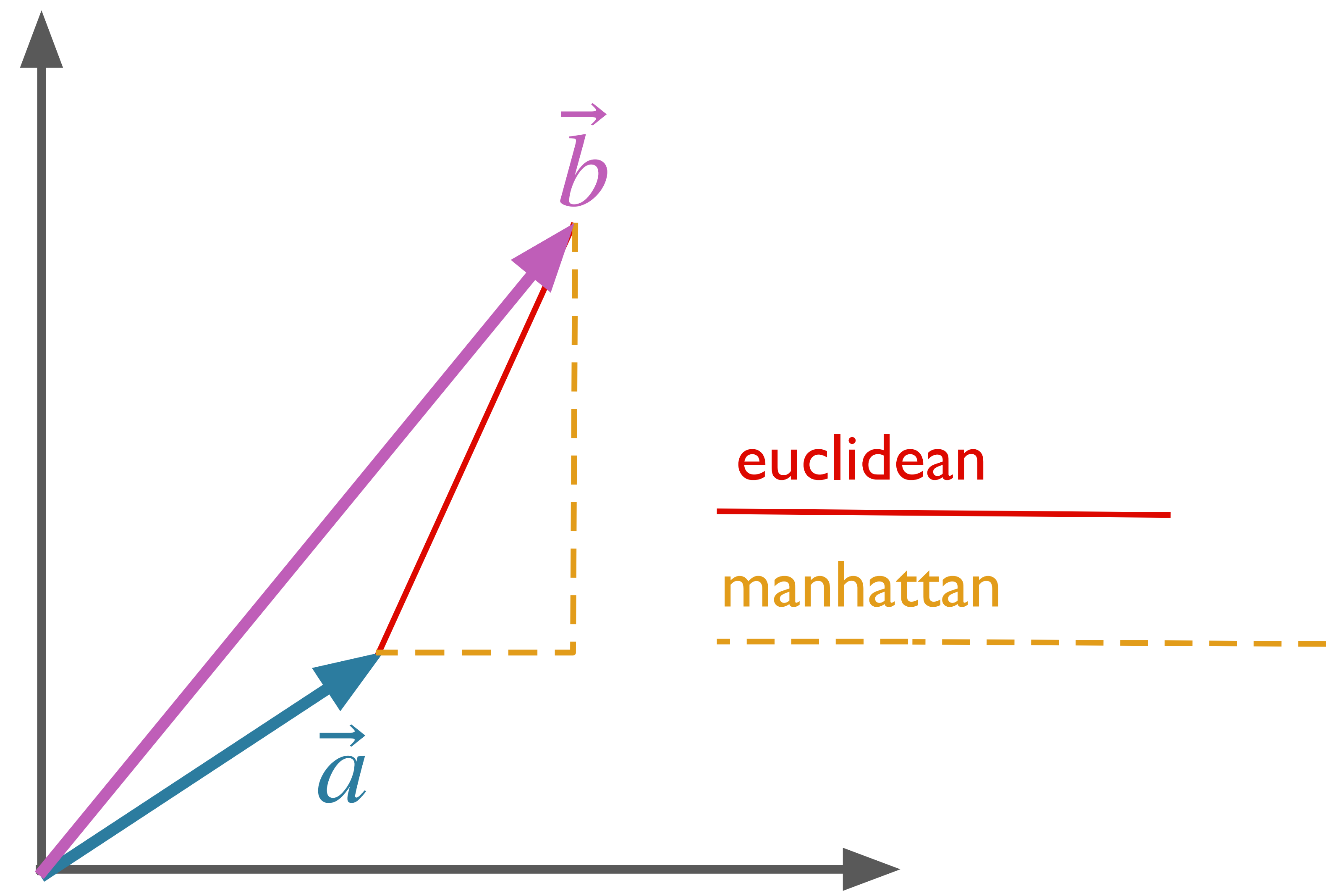

Vector Similarity: Dot Product

- Produces real number scalar from product of vectors' components

$$\text{sim}_{\text{dot}}(x, y) = x \cdot y = \sum_i x_i \times y_i$$

- Biased toward ***longer*** (larger magnitude) vectors
  - In our case, vectors with fewer zero counts

Vector Similarity: Cosine

- If you normalize the dot product for vector magnitude…
- …result is same as cosine of angle between the vectors.

$$\text{sim}_{\text{cos}}(x, y) = \frac{x \cdot y}{\|x\| \, \|y\|} = \frac{\sum_i x_i \times y_i}{\sqrt{\sum_i x_i^2}\sqrt{\sum_i y_i^2}}$$

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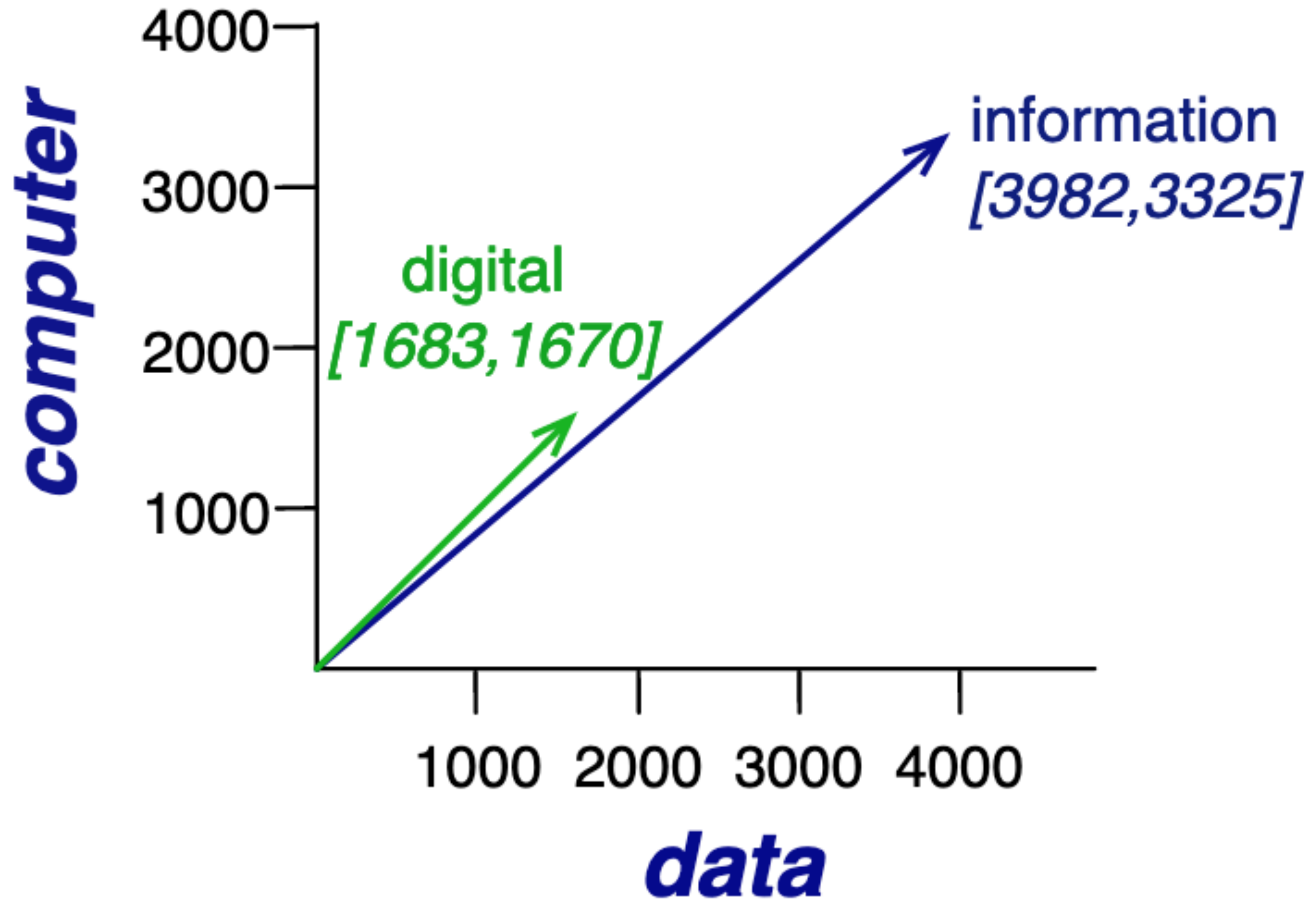
  - 'Company' = context
- Word represented by context feature vector
  - Many alternatives for vector
- Initial representation:
  - 'Bag of words' feature vector
  - Feature vector length N, where N is size of vocabulary
    - f_i+=1 if $word_i$ within window size w of $word$

Bag of Words Vectors

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	...
strawberry	0	...	0	0	1	60	19	...
digital	0	...	1670	1683	85	5	4	...
information	0	...	3325	3982	378	5	13	...



Bag of Words Vectors

- Usually re-weighted, with e.g. tf-idf, ppmi
- Still sparse
- Very high-dimensional: |V|

	aardvark	...	computer	data	result	pie	sugar	...
cherry	0	...	2	8	9	442	25	...
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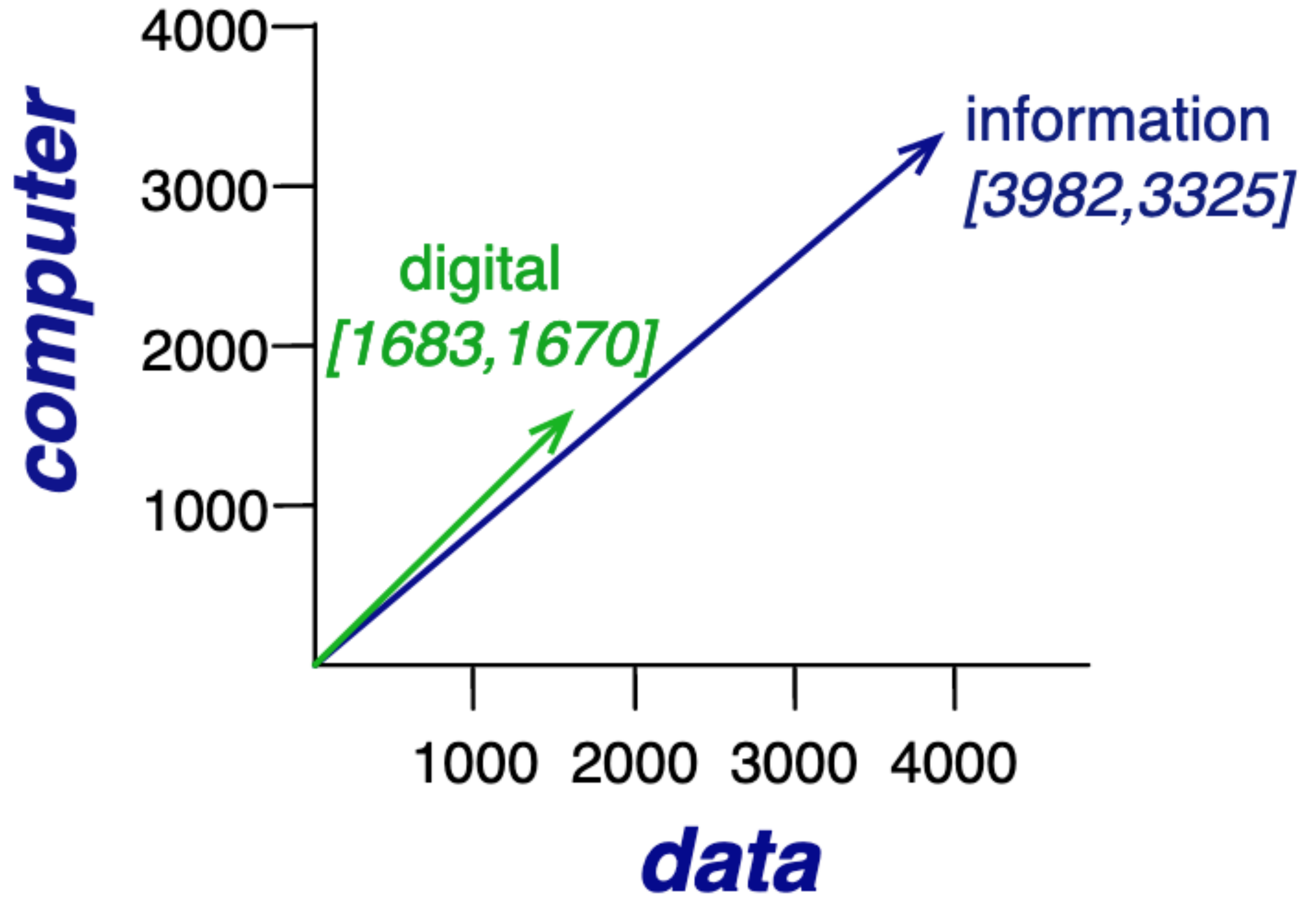

Homework 1

Next Time

- Skip-Gram with Negative Sampling
  - How optimization framework applies to this problem
- Introduction of two tasks that we will use throughout the class
  - Language modeling
  - Text classification [sentiment analysis]