Computability and Logic

MWF, 9:30 - 10:20am @ Cubberley 334

Contact Information

Shane Steinert-Threlkeld

shanest@stanford.edu

Office Hours: Wednesday, 10:30am - 12pm in 90-92H (or by appointment)

Course Website: on Canvas

Description

The main goal of this course will be to prove Gödel's two Incompleteness Theorems. These landmark results state that any sufficiently strong formal theory will have true-but-unprovable sentences and cannot prove its own consistency. To get there, we will look at the expressivity of various formal theories of arithmetic, different approaches to effective computation (recursive functions, Turing machines), and the connection between the two.

Reading Material

We will work through Chapter 3 of the following book, which is required:

- Herbert B Enderton, *A Mathematical Introduction to Logic*, Second Edition, Academic Press, 2001.

I will supplement this book on occasion. When necessary, additional required readings will be made available on the Canvas website.

The following books are recommended for supplementing the technical material:

- Boolos, Burgess, and Jeffrey, *Computability and Logic*.
- Smith, *An Introduction to Gödel's Theorems*, second edition.
- For other books on recursion theory, first-order logic, and the like, ask the instructor.

Other books for further reading:

- Franzen, *Godel's Theorem: An Incomplete Guide to its Use and Abuse*
- Penrose, *Shadows of the Mind*

All of these will be on reserve in Tanner Library, in Building 90.

Requirements / Grading

The workload for the course will be three problem sets and a take-home final exam. You may discuss and collaborate on the homeworks, but must write up the solutions on your own. The final exam must be done alone. Solutions will be made available in Tanner Library. The grading breakdown will be:

- Homeworks 1-3: 20% each

- Final exam: 30%
- Attendance and participation: 10%

Disability Accommodation

Students who have a disability which may necessitate an academic accommodation or the use of auxiliary aids and services in a class must initiate the request with the Office of Accessible Education (OAE). The OAE will evaluate the request with required documentation, recommend appropriate accommodations, and prepare a verification letter dated in the current academic term in which the request is being made. Please contact the OAE as soon as possible: timely notice is needed to arrange for appropriate accommodations. The OAE's contact details are as follows.

Address: 563 Salvatierra Walk, Stanford, CA 94305

Phone: (650) 723-1066

Web address: http://studentaffais.stanford.edu/oae

Schedule

03/28 Introduction and Motivation

03/30 Basic Concepts

- Sections 1.7, 2.6, 3.0 of Enderton

04/01 Class Cancelled (at a conference)

Decidable Fragments

04/04 Natural numbers with successor

04/06 Natural numbers with successor **HW1 out**

- Section 3.1 of Enderton

04/08 Successor and Order

04/11 Successor and Order

04/13 Successor and Order

- Section 3.2 of Enderton

A Sufficiently Strong Arithmetic

04/15 Successor, Order, Multiplication, and Exponentiation **HW1 due**

04/18 Representability

04/20 Representability in A_E

- Section 3.3 of Enderton

04/22 Arithmetization of Syntax

04/25 Arithmetization of Syntax

04/27 Arithmetization of Syntax **HW2 out**

- Section 3.4 of Enderton

Incompleteness and Undecidability

04/29 First Incompleteness Theorem + Undefinability of Truth

05/02 Incompleteness and Undecidability

- Section 3.5 of Enderton

05/04 Second Incompleteness Theorem

05/06 Second Incompleteness Theorem **HW2 due**

- Section 3.7 of Enderton

05/09 Philosophical Consequences

- Peter Koellner, "On the Question of whether the Mind can be Mechanized"

05/11 Johan van Benthem: Provability Logic **HW3 out**

Turing Machines and Recursive Functions

05/13 (Universal) Turing Machines

05/16 Undecidability of the Halting Problem

- Chapters 3 and 4 of Boolos, Burgess, and Jeffrey

05/18 (Primitive) Recursive Functions

05/20 Recursive Functions **HW3 due**

- Section 3.6 of Enderton

Overcoming the Limitations

05/23 Tenenbaum's Theorem

- §25.2 of Boolos, Burgess, and Jeffrey

05/25 Second-Order Logic **Final exam out**

05/27 Second-Order Logic

- Section 4.1 of Enderton

05/30 Memorial Day: NO CLASS

06/01 Review and Discussion

06/03 **Final exam due**